

Paper Reference 9FM0/4D
Pearson Edexcel Level 3 GCE

Further Mathematics
Advanced
PAPER 4D: Decision Mathematics 2

Monday 24 June 2024 – Afternoon

Time: 1 hour 30 minutes

YOU MUST HAVE

**Mathematical Formulae and Statistical Tables (Green),
calculator**

YOU WILL BE GIVEN

**Diagram Booklet
Answer Booklet**

X72803A

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Do not return this Question Paper with the Answer Booklet.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or on the separate diagrams – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

**There are 8 questions in this Question Paper.
The total mark for this paper is 75.**

**The marks for EACH question are shown in brackets
– use this as a guide as to how much time to spend on
each question.**

**There are two copies of each diagram in case you need
them.**

ADVICE

**Read each question carefully before you start to answer
it.**

Try to answer every question.

Check your answers if you have time at the end.

1. Refer to Diagram 1 and Diagram 2 in the Diagram Booklet.

Diagram 1 shows a capacitated, directed network of pipes.

The numbers in circles represent an initial flow from **S** to **T**

The other number on each arc represents the capacity, in litres per second, of the corresponding pipe.

(a) (i) State the value of **x**

(ii) State the value of **y**

(2 marks)

(b) State the value of the initial flow.

(1 mark)

(c) State the capacity of cut **C₁**

(1 mark)

(continued on the next page)

Turn over

1. continued.

(d) Find, by inspection, a flow–augmenting route to increase the flow by four units.

You must state your route.

(1 mark)

The flow–augmenting route from (d) is used to increase the flow from **S** to **T**

(e) Prove that the flow is now maximal.

(3 marks)

A vertex restriction is now applied so that no more than **12** litres per second can flow through **E**

(f) (i) Complete Diagram 2 in the Diagram Booklet to show this restriction.

(ii) State the **VALUE** of the maximum flow through the network with this restriction.

(2 marks)

(Total for Question 1 is 10 marks)

2. The general solution of the first order recurrence relation

$$u_{n+1} + au_n = bn^2 + cn + d \quad n \geq 0$$

is given by

$$u_n = A(3)^n + 5n^2 + 1$$

where **A** is an arbitrary non-zero constant.

By considering expressions for u_{n+1} and u_n , find the values of the constants **a**, **b**, **c** and **d**

(Total for Question 2 is 3 marks)

- 3. Refer to Diagram 3, Diagram 4 and Diagram 5 in the Diagram Booklet.**

Diagram 3 shows the cost, in pounds, of transporting one unit of stock from each of four supply points, E, F, G and H, to three sales points, A, B and C

It also shows the stock held at each supply point and the amount required at each sales point.

A minimum cost solution is required.

- (a) Explain why it is necessary to add a dummy demand point.**

(1 mark)

- (b) On Diagram 4 in the Diagram Booklet, insert appropriate values in the dummy demand column, D**

(1 mark)

(continued on the next page)

3. continued.

Diagram 5 shows what the table becomes after finding an initial feasible solution and applying one iteration of the stepping–stone method.

(c) Starting with **GD as the next entering cell, perform two further iterations of the stepping–stone method to obtain an improved solution.**

You must make your method clear by showing your routes and stating the

- shadow costs**
- improvement indices**
- entering and exiting cells**

(6 marks)

(continued on the next page)

3. continued.

(d) State the cost of the solution found in (c).

(1 mark)

(e) Determine whether the solution obtained in (c) is optimal, giving a reason for your answer.

(3 marks)

(Total for Question 3 is 12 marks)

4. Refer to Diagram 6 in the Diagram Booklet.

Four workers, **A**, **B**, **C** and **D**, are to be assigned to four tasks, **P**, **Q**, **R** and **S**

Each task must be assigned to just one worker and each worker can do only one task.

Worker **B** cannot be assigned to task **Q** and worker **D** cannot be assigned to task **R**

The amount, in pounds, that each worker would earn when assigned to each task is shown in Diagram 6

(continued on the next page)

4. continued.

The Hungarian algorithm can be used to find the maximum total amount that would be earned by the four workers.

(a) (i) Explain how to modify the table so that the Hungarian algorithm could be applied.

(ii) Modify the table as described in (a)(i).

(3 marks)

(b) Formulate the above situation as a linear programming problem.

You must define the decision variables and make the objective function and constraints clear.

(6 marks)

(Total for Question 4 is 9 marks)

5. Refer to Diagram 7 in the Diagram Booklet.

Sebastien needs to make a journey.

He can choose between travelling by plane, by train or by coach.

Sebastien knows the exact costs of all three travel options, but he also wants to account for his travel time, including any possible delays.

The cost of Sebastien's time is £50 per hour.

Diagram 7 shows the costs, the journey times (without delays), and the corresponding probabilities of delays, for each travel option.

(a) By drawing a decision tree, evaluate the EMV of the total cost of Sebastien's journey for each node of your tree.

(6 marks)

(continued on the next page)

5. continued.

(b) Hence state the travel option that minimises the EMV of the total cost of Sebastien's journey.

(1 mark)

(c) A cube root utility function is applied to the total costs of each option.

Determine the travel option with the best expected utility and state the corresponding value.

(3 marks)

(Total for Question 5 is 10 marks)

6. Refer to Diagram 8 and Diagram 9 in the Diagram Booklet.

The staged, directed network in Diagram 8 represents the roads that connect 12 towns, S, A, B, C, D, E, F, G, H, I, J and T

The number on each arc shows the time, in hours, it takes to drive between these towns.

Elena plans to drive from S to T
She must arrive at T by 9 pm

- (a) By completing Diagram 9 in the Diagram Booklet, use dynamic programming to find the latest time that Elena can start her journey from S to arrive at T by 9 pm
(9 marks)

- (b) Hence write down the route that Elena should take.
(1 mark)

(Total for Question 6 is 10 marks)

7. Refer to Diagram 10 and Diagram 11 in the Diagram Booklet.

A two person zero–sum game is represented by the pay–off matrix for player **A**, shown in Diagram 10.

- (a) Verify that there is no stable solution to this game.

(2 marks)

(continued on the next page)

7. continued.

Player **A** intends to make a random choice between options **R**, **S** and **T**, choosing option **R** with probability p_1 , option **S** with probability p_2 and option **T** with probability p_3

Player **A** wants to find the optimal values of p_1 , p_2 and p_3 using the Simplex algorithm.

Player **A** formulates the following objective function for the corresponding linear programme.

Maximise $P = V$ where $V = \text{the value of the game} + 3$

(b) Determine an initial Simplex tableau, making your variables and working clear.

The incomplete Simplex tableau is on page 48 of the Answer Booklet.

(5 marks)

(continued on the next page)

7. continued.

After several iterations of the Simplex algorithm, a possible final tableau is shown in Diagram 11.

(c) (i) State the best strategy for player A

(ii) Calculate the value of the game for player B

(3 marks)

Player B intends to make a random choice between options X, Y and Z

(d) Determine the best strategy for player B, making your method and working clear.

(3 marks)

(Total for Question 7 is 13 marks)

8. A sequence $\{u_n\}$, where $n \geq 0$, satisfies the recurrence relation

$$2u_{n+2} + 5u_{n+1} = 3u_n + 8n + 2$$

- (a) Find the general solution of this recurrence relation.

(5 marks)

A particular solution of this recurrence relation has $u_0 = 1$ and $u_1 = k$, where k is a positive constant. All terms of the sequence are positive.

- (b) Determine the value of k

(3 marks)

(Total for Question 8 is 8 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
